Modeling a 3D Eddy Current Problem Using the Weak Formulation of the Convective A*- ϕ Steady State Method

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1. Introduction

A 3D model of a magnetic (Halbach) rotor both rotating and translationally moving at high-speed over a conductive (non-magnetic) guideway is modeled in steady-state using the convective $\mathbf{A}^*$- ϕ formulation [1-3]. An illustration of the problem to be modeled is shown in Figure 1. The formulation is implemented in COMSOL using the weak formulation. The presence of the magnetic rotor (source field) is incorporated into the formulation via the boundary conditions. This type of problem is difficult to model using existing commercial packaged electromagnetic software. In order to validate the COMSOL model the calculated results are compared with experimental measurements.

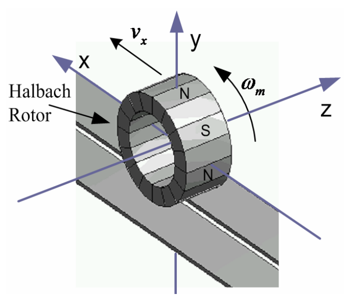

Figure 1: A rotor both rotating and translationally moving, at high-speed, over a conductive guideway

2. Use of Weak Form of COMSOL Multiphysics

The Weak Formulation of COMSOL was utilized. A 2D illustration of the problem domain is shown in Figure 2. The problem formulation within the conducting, Ω_c, and non-conducting regions, Ω_{nc}, as well as boundary conditions are briefly outlined in the following three sections.

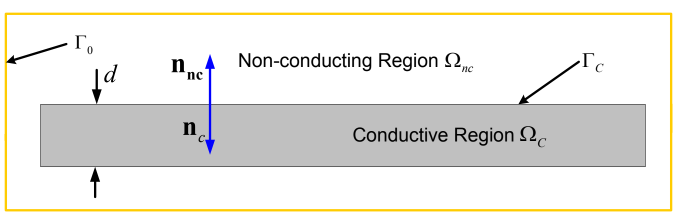

Figure 2: 2D schematic of the finite element model.

2.1 Conducting Region

If the conductors are assumed to move, rather than the magnets, then the applicable quasi-static Maxwell's equations and vector potential relations within the conductors are [4]

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \tag{1}$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} \tag{2}$$

$$\nabla \cdot \mathbf{B} = 0 \tag{3}$$

$$\mathbf{J} = \sigma(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \quad (4)$$

$$\mathbf{B} = \nabla \times \mathbf{A} \quad (5)$$

$$\mathbf{E} = -\frac{\partial \mathbf{A}}{\partial t} - \nabla V \quad (6)$$

where σ = guideway conductivity [Sm^{-1}]
$\mathbf{v}$ = velocity [ms^{-1}]
μ_0 = permeability of free space [Hm^{-1}]

If the guideway material is simply connected, linear, homogeneous and composed of non-magnetic material, such as aluminum, then using (1)-(6) and the gauge condition:

$$\nabla \cdot \mathbf{A} = 0 \quad (7)$$

enables the governing guideway equation to be expressed solely in terms of the magnetic vector potential [1-3]

$$\nabla^2 \mathbf{A} = \mu_0 \sigma \left(\frac{\partial \mathbf{A}}{\partial t} + \left(\mathbf{v} \cdot \nabla \right) \mathbf{A} \right), \text{ in } \Omega_c, \quad (8)$$

Assuming a steady-state solution can be obtained in which

$$\mathbf{A}(x, y, z, t) = \mathbf{A}(x, y, z) e^{j\omega_e t} \text{ , in } \Omega_c \quad (9)$$

where ω_e is the electric angular velocity, then (8) will become

$$\nabla^2 \mathbf{A} = \mu_0 \sigma \left(j\omega_e \mathbf{A} + v_x \frac{\partial \mathbf{A}}{\partial x} \right), \text{ in } \Omega_c \quad (10)$$

where (10) assumes that the velocity only has an x-component. The three individual vector potential components in (10) have the same weak form. Using the Galerkin weighted residual procedure (10) can be rewritten, using Green's first identity, as [1]

$$-\int_{\Omega_c} \nabla N_n \cdot \nabla A_n d\Omega_c - \mu_0 \sigma \int_{\Omega_c} N_n \left(v_x \frac{\partial A_n}{\partial x} + j\omega_e A_n \right) d\Omega_c + \int_{\Gamma_c} N_n \left(\nabla A_n \cdot \mathbf{n}_c \right) d\Gamma_c = 0, \quad (11)$$

where $n = x,y,z$ and N_n is the n-component shape function. This weak formulation was entered into COMSOL. The relationship between the mechanical, ω_m, and electrical, ω_e, rotor frequency will be

$$\omega_e = \frac{P}{2} \omega_m \quad (12)$$

where P is the number of poles.

2.2 Non-Conducting Region
Within the non-conducting region, in which magnets are present, the total field intensity can be expressed as

$$\mathbf{H} = \frac{\mathbf{Brotor}}{\mu_0} - \nabla\phi \text{ , in } \Omega_{nc} \tag{13}$$

Where ϕ is the magnetic scalar potential due to the induced guideway currents and $\mathbf{Brotor}$ is Halbach complex magnetic rotors field (flux density). This is calculated using an analytic based formula, *the derivation of which is discussed in the full paper*. If the magnetic material is linear then after taking the divergence of both sides of (13) the formulation for the air region will be

$$\nabla^2\phi = 0 \text{ , in } \Omega_{nc} \tag{14}$$

Therefore, it is not necessary to explicitly model the rotor's field within the non-conducting region if its effect is accounted for by the guideway boundary conditions [5, 6]. After using Green's first identity the weak form of (14) will be

$$\int_{\Omega_{nc}} \nabla\phi \cdot \nabla w_f d\Omega_{nc} - \int_{\Gamma_{nc}} w_f \nabla\phi \cdot \mathbf{n}_{nc} d\Gamma_c = 0 \tag{15}$$

where w_f is the weighting function. This weak formulation was entered into COMSOL.

2.3 Boundary Conditions
The guideway interface conditions for the normal and tangential field components at the $\mathbf{A} - \phi$ guideway interface are [6-8]

$$-\mu_0 \nabla\phi \cdot \mathbf{n}_{nc} + \mathbf{Brotor} \cdot \mathbf{n}_{nc} = \nabla \times \mathbf{A} \cdot \mathbf{n}_{nc} \text{ , on } \Gamma_c \tag{16}$$

$$-\mathbf{n}_c \times \mu_0 \nabla\phi + \mathbf{n}_c \times \mathbf{Brotor} = \mathbf{n}_c \times \nabla \times \mathbf{A} \text{ , on } \Gamma_c \tag{17}$$

In addition, in order to ensure the uniqueness of the solution

$$\mathbf{n}_c \cdot \mathbf{A} = 0 \text{ , on } \Gamma_c \tag{18}$$

must also be enforced on the conductive boundaries [5, 9, 10]. This also sets the normal component of the current on the boundaries to zero ($\mathbf{n}_c \cdot \mathbf{J} = 0$). The field due to the magnetic rotor was included in the weak formulation by modifying the boundary conditions given in (11) and (15). *The details are discussed in the full paper.* The Dirichlet boundary condition has been applied on all of the remaining non-conducting boundaries

$$\phi_1 = 0, \text{ on } \Gamma_0 \tag{19}$$

3. Results
An example of the iso-surface plot in the non-conducting region due to the induced guideway currents created when the rotor is both rotated and translationally moved is illustrated in Figure 3. In this case the rotor is offset from the center. Figure 4 shows the A_z field on the surface of the conductive region when the rotor is rotated over one of the conducting guideway sheets. The model was written in COMSOL ver. 3.1

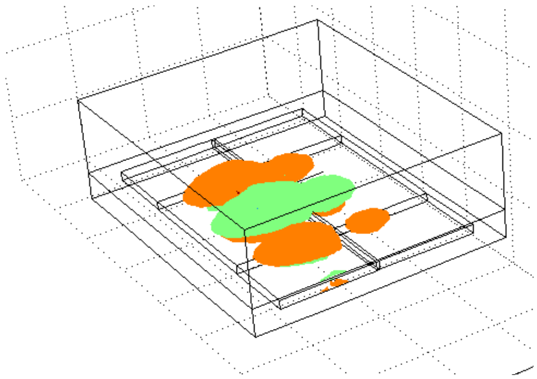

Figure 3: Perspective view of a B_x magnetic flux density iso-surface plot in the non-conducting region due to the induced guideway currents

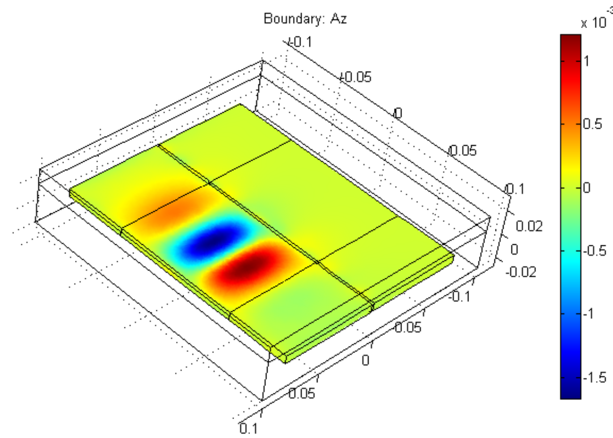

Figure 4: The A_z field on the surface of the conductive region is shown.

4. Conclusion

The abstract presents an overview of a novel steady-state convective $\mathbf{A}^*$- ϕ formulation that was implemented successfully using COMSOL's weak form. The magnetic source within the non-conducting region was not explicitly modeled using finite elements because its presence was modeled by incorporating analytic (complex) source field terms into the boundary conditions between the conducting and non-conducting regions. This formulation was verified by comparing it with experimental results. The comparison with experimental results and the full weak-form boundary condition interface formulation will be provided in the full paper.

5. References

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